

# Multi-task group Lasso for Genome Wide Association Studies in diverse populations

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October 25th 2021



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**PhD advisor:** Chloé-Agathe Azencott

### **Background:**

Engineer  
Master's degree  
Machine Learning



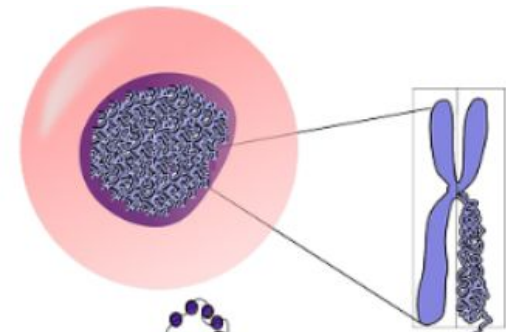
Chloé-Agathe Azencott



Thomas Walter, Director of CBIO



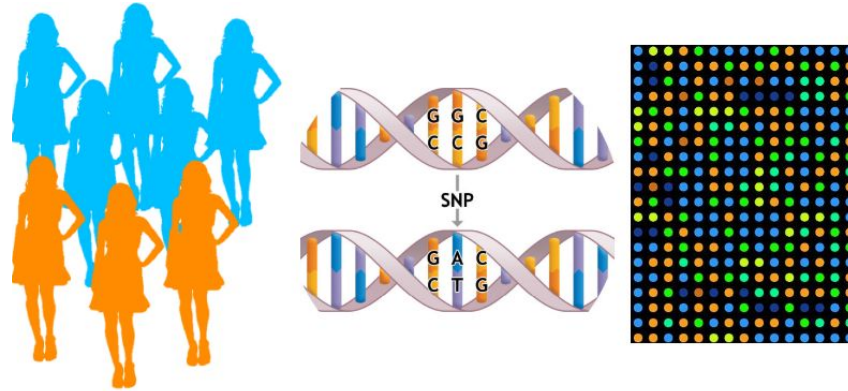
# Genome Wide Association Studies



- The Human Genome Project (HGP) provides a good mapping to decode the whole genome at Single Nucleotide Variant (SNVs) level.
- Single Nucleotide Polymorphism (SNPs) are common SNVs at a frequency of 1%.
- 3 billions base pair divided in 24 chromosomes.
- 15 millions SNPs.
- Find association between genome and disease risk.



# Genome Wide Association Studies



Goal: Find association between the genotype and the phenotype.

- The genotype: Single Nucleotide Polymorphism (SNP) arrays.
- The phenotype:
  - Quantitative: BMI, weight, height, etc.
  - Qualitative: Case-control study

# Challenges in GWAS analysis

01

## Single marker analysis

Testing each SNP individually

02

## Population stratification

Difference in allele frequencies between subpopulations

03

## Linkage disequilibrium

Dependence relationship between two alleles at two different loci

04

## Computational limitations

For complex methods:  
- Memory errors  
- Very slow

05

## Lack of stability

Susceptibility to small perturbations in the data set

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individually

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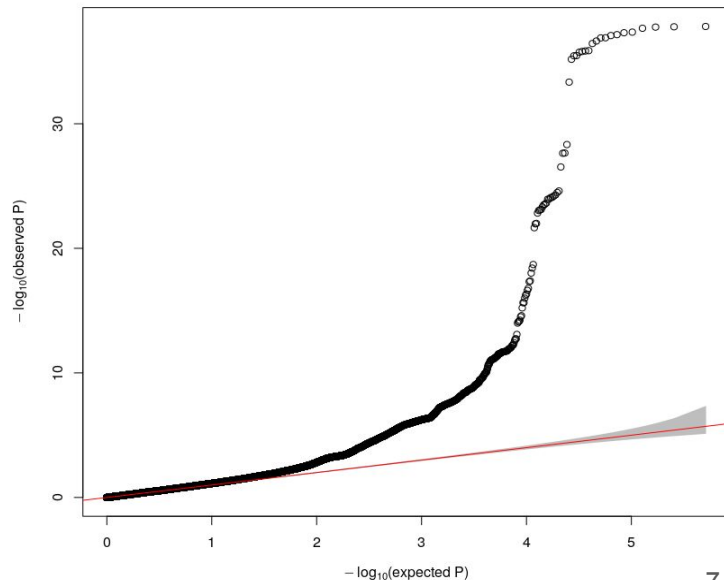
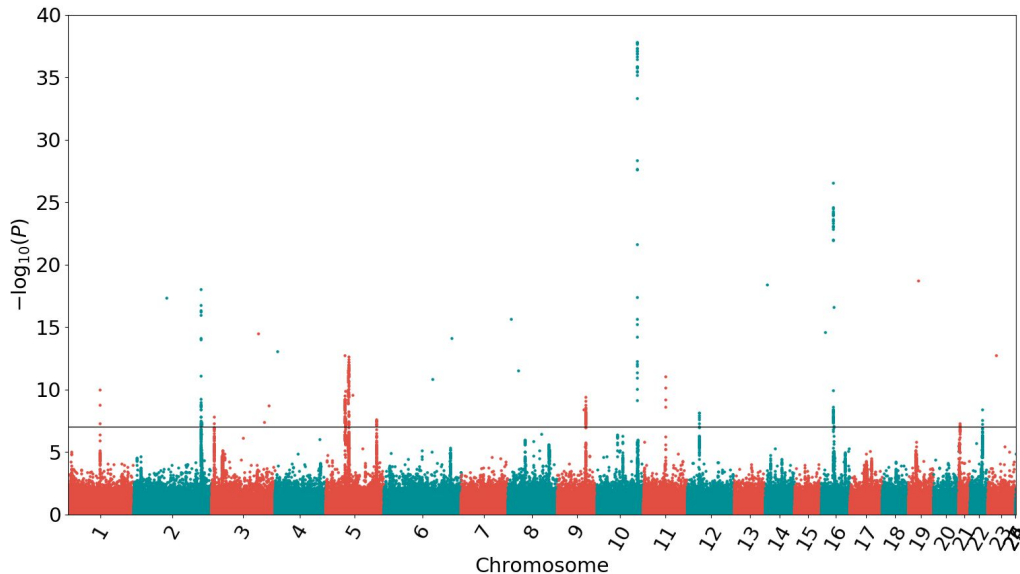
Susceptibility to small perturbations in the data set

# From GWAS to Machine Learning

- **Single-marker analysis:**

Given a phenotype  $\mathbf{y}$ ,  $\mathbf{X}$  is the genotype matrix:

For each feature  $\mathbf{X}_j$ , we fit a **single-predictor** equation  $\mathbf{y} = \beta_0 + \beta_j \mathbf{X}_j + \varepsilon \Rightarrow$  **p-value from a t-test** against an intercept-only model  $H_0 = \{\beta_j = 0\}$ .



# From GWAS to Machine Learning

- **Multi-variate approach:** Feature selection based on regularization

- **Regularization:** adding a penalty term

$$\operatorname{argmin}_{\beta \in \mathbb{R}^p} \underbrace{\mathcal{L}(y, \beta X_j)}_{\text{loss}} + \underbrace{\lambda \Omega(\beta_1, \beta_2, \dots, \beta_j)}_{\text{regularization term}}$$

- **Lasso:** shrinkage and feature selection (L1-regularization)

$$\operatorname{argmin}_{\beta \in \mathbb{R}^p} \underbrace{\mathcal{L}(y, \beta X_j)}_{\text{loss}} + \underbrace{\lambda \sum_{j=1}^p |\beta_j|}_{\text{sparsity}}$$

- **Group lasso:** allow predefined groups of covariates to be jointly selected

$$\operatorname{argmin}_{\beta \in \mathbb{R}^p} \underbrace{\mathcal{L}(y, \beta X_j)}_{\text{loss}} + \underbrace{\lambda \sum_{g \in \mathcal{G}} \sqrt{p_g} \|\beta_g\|_2}_{\text{sparsity at the group level}}$$

- **Multi-task lasso:** allows fitting multiple regression problems jointly

$$\operatorname{argmin}_{\beta \in \mathbb{R}^{T \times p}} \underbrace{\sum_{t=1}^T \frac{1}{n_t} \sum_{m=1}^{n_t} \mathcal{L}\left(y^{(tm)}, \left(\beta_0^{(t)} + \sum_{j=1}^p \beta_j^{(t)} X_j^{(tm)}\right)\right)}_{\text{loss}} + \underbrace{\lambda \sum_{j=0}^p \sum_{t=1}^T |\beta_j^{(t)}|}_{\text{task sharing}}$$

where  $\beta$  is a  $p \times 1$  vector corresponds to the SNP effects

$\Omega$  is the regularizer

$\lambda$  is the penalization term

where  $\mathcal{G}$  is the set of groups

$\beta_g$  is  $\beta$  restricted to the SNPs in  $g$

$\sqrt{p_g}$  scales the penalization factor according to the group size

where  $T$  is the number of tasks to learn the training set

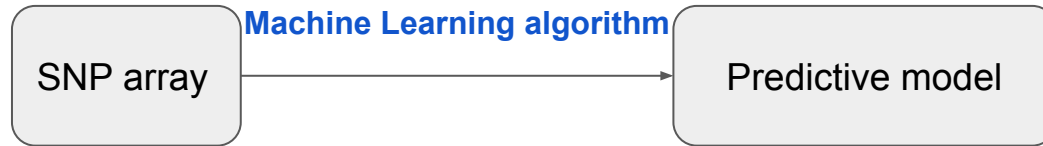
$\{(x_{tm}, y_{tm}) \text{ for } t=1..T \text{ and } m=1..n_t\}$



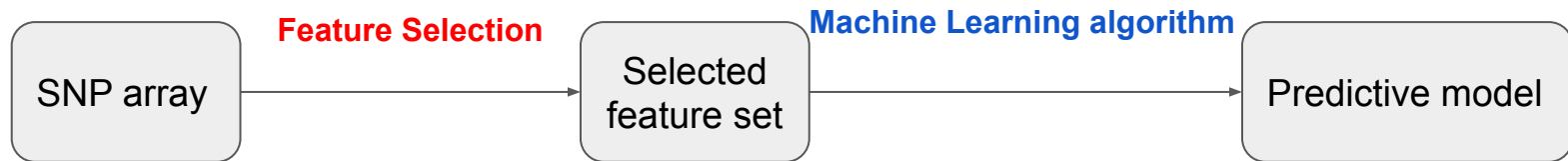
# From GWAS to Machine Learning

- **Microarray data: SNP arrays**

Curse of dimensionality ( $p \gg N$ ):  $p \approx 10^5 - 10^7$ ,  $N \approx 10^2 - 10^4$



- **Biomarker identification : feature selection models**



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# Population stratification

**Population stratification** refers to the presence of differences in allele frequencies between subpopulations due to different ancestry.

## ● State-of-the art adjustment methods

### ● PCA-based methods

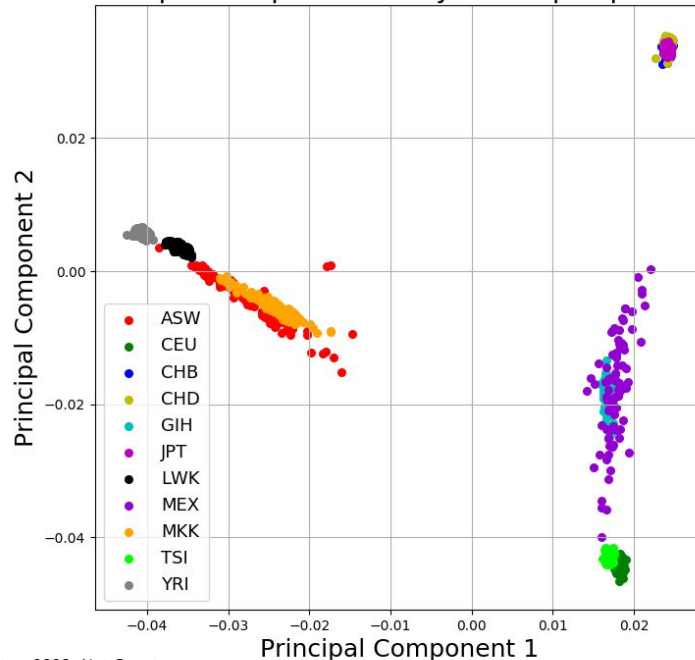
Include Principal components (PCs) as covariates

- Logistic Regression + Top PCs<sup>[1,2]</sup>
- EIGENSTRAT<sup>[3]</sup>: multi-linear regression + 10 PCs

### ● Linear mixed models

Fast-LMM<sup>[4]</sup>

Principal Component Analysis - HapMap3 data



<sup>[1]</sup> Need et al., A genome-wide investigation of snps and cnvs in schizophrenia. 2009, *PLoS Genet.*

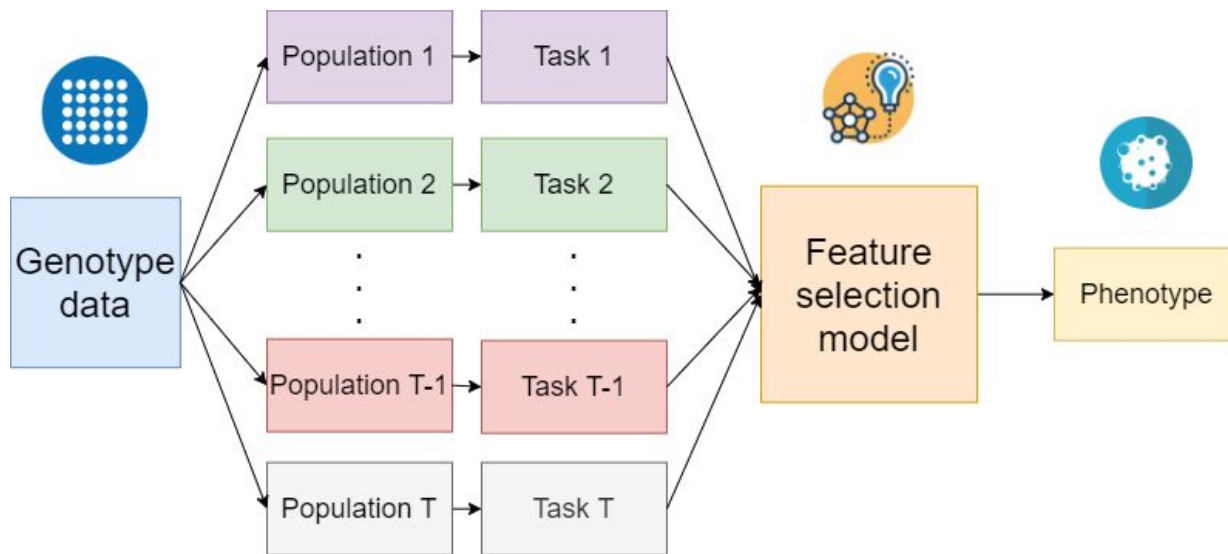
<sup>[2]</sup> Zeggini et al., Meta-analysis of genome-wide association data and large-scale replication identifies additional susceptibility loci for type 2 diabetes. 2008, *Nat Genet.*

<sup>[3]</sup> Price et al., Principal components analysis corrects for stratification in genome-wide association studies. *Nat Genet.*

<sup>[4]</sup> Lippert et al., FaST linear mixed models for genome-wide association studies. 2011. *Nat Methods.*

# Population stratification

- **Proposed adjustment method:** subpopulations assignment in multitask framework



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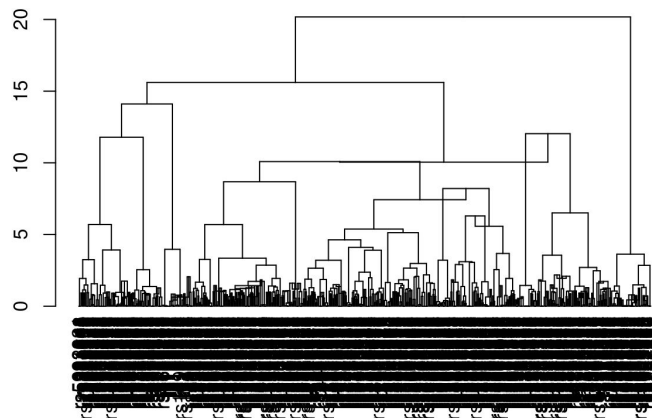
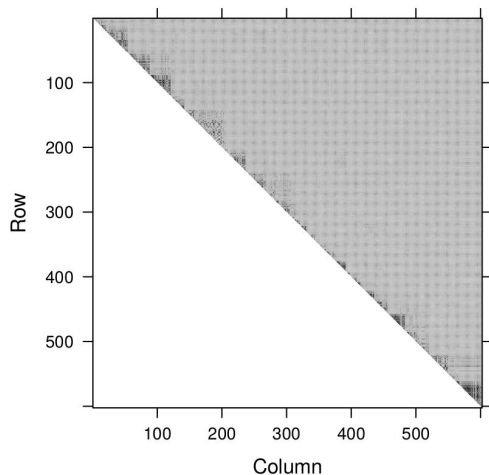
# Linkage Disequilibrium groups clustering

## Linkage Disequilibrium (LD):

- Tendency of alleles to be transmitted together, more often than expected by chance alone.
- Usually caused by close proximity of genes in the same chromosome.

## Hierarchical clustering approach<sup>[1]</sup>

Performing a **spatially-constrained hierarchical clustering**



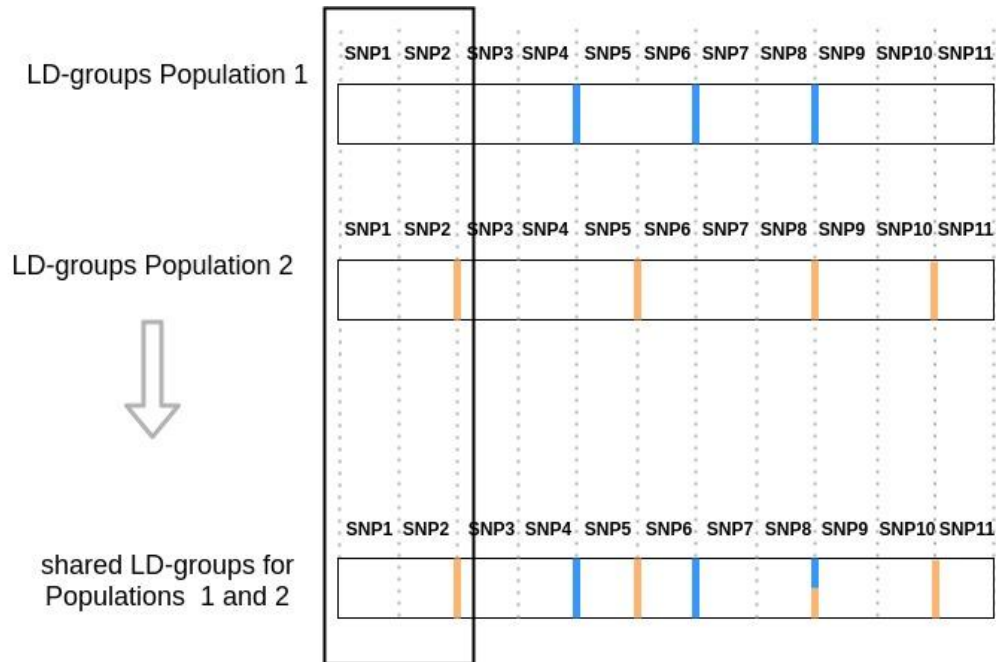
⇒ Selection at the **LD-group level** instead of single-SNP level.

<sup>[1]</sup> Ambroise et al., Adjacency-constrained hierarchical clustering of a band similarity matrix with application to genomics. 2019. arXiv:1902.01596v1 [math.ST].

# Linkage Disequilibrium groups clustering

- **Choice of LD-groups**

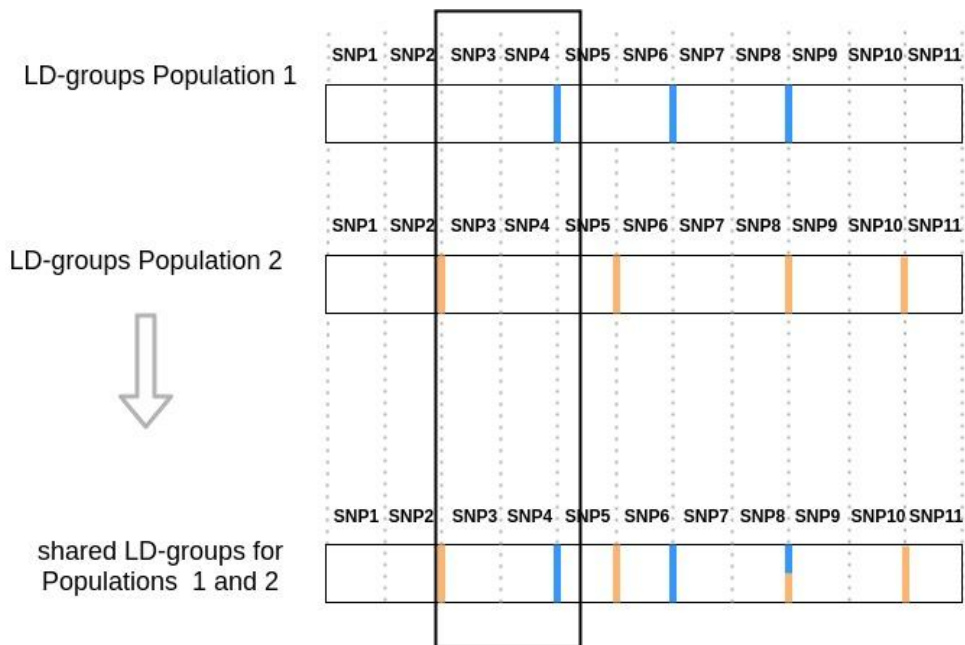
Linkage disequilibrium is different in different populations



# Linkage Disequilibrium groups clustering

- **Choice of LD-groups**

Linkage disequilibrium is different in different populations

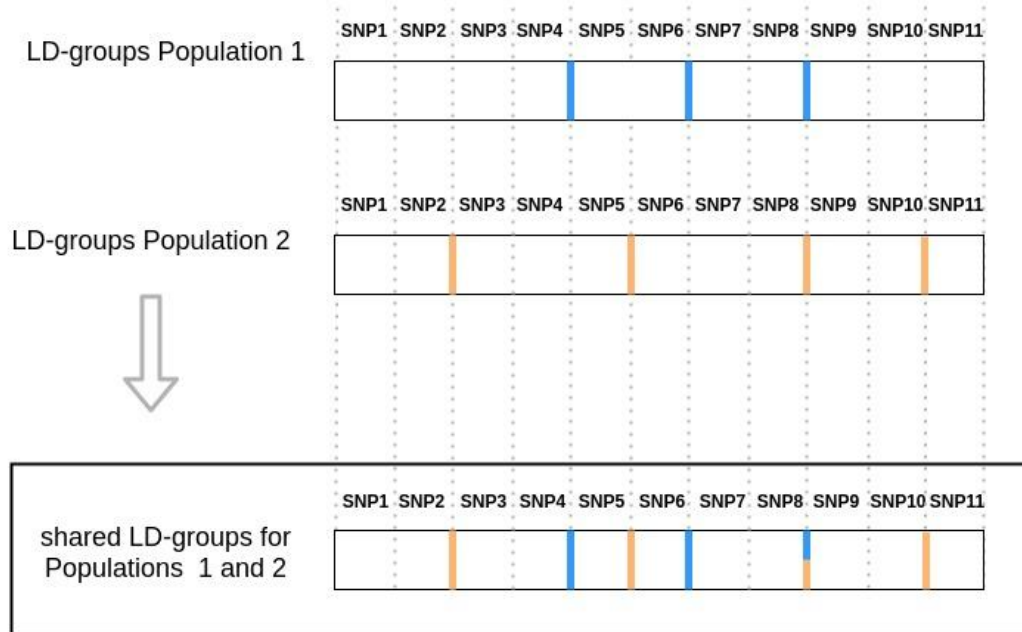




# Linkage Disequilibrium groups clustering

- **Choice of LD-groups**

Linkage disequilibrium is different in different populations



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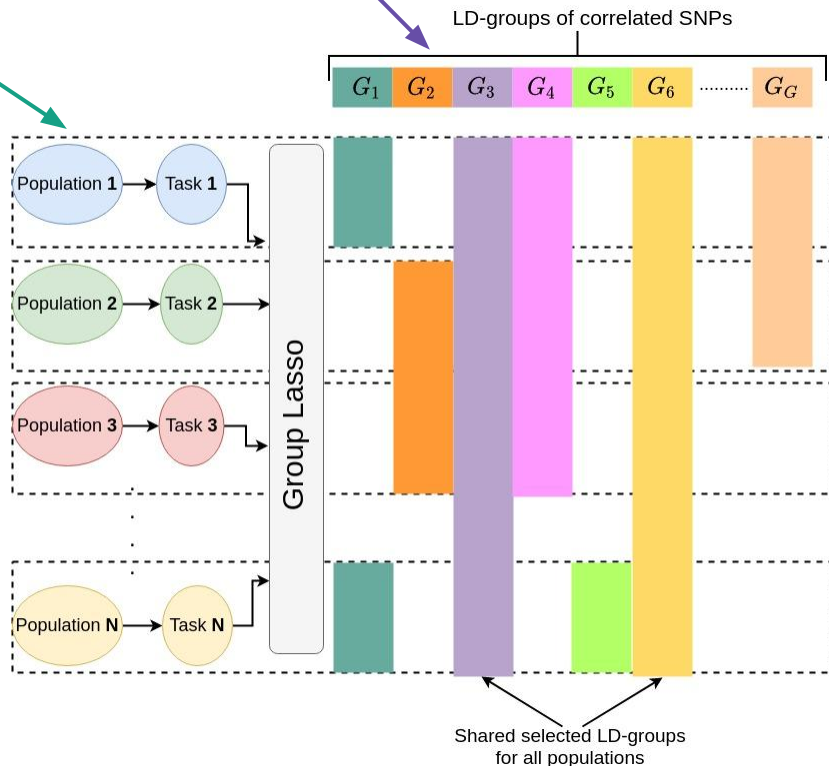
# Multitask group Lasso for Genome Wide Association studies in diverse populations

Multitask group Lasso where **tasks** correspond to **subpopulations** and **groups** correspond to **LD-groups** of strongly correlated SNPs

$$\min_{B \in \mathbb{R}^{T \times (p+1)}} \underbrace{\sum_{t=1}^T \frac{1}{n_t} \sum_{m=1}^{n_t} \mathcal{L} \left( y^{(tm)}, \left( \beta_0^{(t)} + \sum_{j=1}^p \beta_j^{(t)} x_j^{(tm)} \right) \right)}_{\text{loss for each task}} + \lambda \underbrace{\sum_{g=1}^G \sqrt{p_g} \|B_g\|_F}_{\text{sparsity at the LD-group level across tasks}}$$

where

- $\beta^{(t)} \in \mathbb{R}^{p+1}$  is a task-specific vector of regression coefficients
- $\mathcal{L}$  is the loss function (quadratic or logistic regression)
- $B_g$  is a  $T \times p_g$  matrix of the regression coefficients, across all tasks  $T$ , for the SNPs of LD-group  $g$
- $\lambda$  is the penalization parameter
- $\sqrt{p_g}$  scales the penalization factor according the group size



⇒ Selection of LD-groups associated with **the phenotype across all tasks/populations**, or **specifically for some tasks/populations**

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# Gap Safe screening rules

**Gap Safe Screening rules**<sup>[1]</sup>: eliminates features with associated coefficients are proved to be zero at the optimum in order to obtain **more speed up** and to **avoid memory errors**.

Ignoring some variables by exploiting geometric properties of the dual formulation of the following optimization problem:

$$\widehat{\beta}^{(\lambda)} \in \operatorname{argmin}_{\beta \in \mathbb{R}^p} P_\lambda(\beta), \text{ for } P_\lambda(\beta) := F(\beta) + \lambda\Omega(\beta) := \sum_{i=1}^n f_i(x_i^\top \beta) + \lambda\Omega(\beta)$$

where  $f_i: \mathbb{R} \mapsto \mathbb{R}$  are convex and differentiable functions and  $\Omega: \mathbb{R}^p \mapsto \mathbb{R}_+$  is a group-decomposable norm:  $\Omega(\beta) = \sum_{g \in \mathcal{G}} \Omega_g(\beta_g)$  with  $\Omega_g$  a norm of  $\mathbb{R}^{n_g}$

**For group Lasso:** the data fitting term is  $F(\beta) = \frac{\mathcal{L}(y, \beta X_j)}{2}$ ,

The  $L1/L2$ -norm is defined by  $\Omega(\beta) = \Omega_w(\beta)$ :

$$\Omega_w(\beta) := \sum_{g \in \mathcal{G}} w_g \|\beta_g\|_2 \quad \text{and} \quad \Omega_w^D(\xi) := \max_{g \in \mathcal{G}} \frac{\|\xi_g\|_2}{w_g}$$

where  $w = (w_g)_{g \in \mathcal{G}}$  are weights satisfying  $w_g > 0$  for all  $g \in \mathcal{G}$  and  $\Omega_w^D(\xi)$  is the dual norm along the regularization path.

<sup>[1]</sup> Ndiaye et al., Gap Safe Screening Rules for Sparsity Enforcing Penalties. 2017, *Journal of Machine Learning Research* 18.

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# Stability Selection

**Stability selection**<sup>[1]</sup>: **bootstrap aggregation** procedure where feature selection is performed repeatedly on bootstrap subsamples, and the results of all repetitions are aggregated. It allows a **precise statement** of the significance of the selected features set and **reduce false positives**.

## Procedure:

- Identify  $S = \{k: \beta_k \neq 0\}$  a set of non-zero inputs of a sparse parameter vector  $\beta$  of observed data  $(X, y)$
- Feature selection is performed on randomly  $|I| = \frac{n}{2}$  of observations, where  $I \subset \{1, \dots, n\}$
- **Selection Path:** Probability of the selection of a feature  $k \in \{1, \dots, p\}$

$$\pi_k^\lambda = Pr^* \left[ k \in \widehat{S}^\lambda(I) \right], \text{ where } \widehat{S}^\lambda(I) \subset \{1, \dots, p\} \text{ denotes the selected features by a subsample } I$$

⇒ Captures random selection within feature selection algorithms

- For a chosen **cut-off**  $\frac{1}{2} \leq \pi_{thre} \leq 1$ , the set of stable features is:

$$\widehat{S}^{stable} = \left\{ k: \pi_k^\lambda \geq \pi_{thre} \right\}$$

⇒ Only variables that are selected **consistently** across all the random halves remain.

<sup>[1]</sup>Meinshausen et al., Stability selection. 2010. *Journal of the Royal Statistical Society Series B-Statistical Methodology*.

# Multitask group Lasso (MuGLasso) implementation

- Datasets

## Realistic simulated data using GWAsimulator<sup>[1]</sup>

- **Dimension:** 4,000 samples x 1,400,000 SNPs
- **Populations:** 2000 European (CEU), 2000 African (YRI)
- **Phenotype:** 1100 CEU cases, 900 CEU controls, 900 YRI cases, 1100 controls.
- **Disease loci:** chromosomes: **2** (located on 1,000-50,000 SNPs), **12** (located on 10-40,000 SNPs), **19** (1000-50,000 SNPs), **21** (10-10,000 SNPs) and **22** (10-2000 SNPs)

## Real data: DRIVE Breast Cancer OncoArray<sup>[2]</sup>

- **Dimension:** 28,281 samples x 528,620 SNPs
- **Phenotype:** 13,846 cases and 14,435 controls
- **Populations:** USA – Uganda – Nigeria – Cameroon – Australia – Denmark

<sup>[1]</sup> DRIVE: GWAsimulator: a rapid whole-genome simulation program. 2008. *Bioinformatics*, Volume 24, Issue 1, January 2008, Pages 140-142. (URL), accessed under project #17707.



# MuGLasso implementation

- Quality control and preprocessing

- MAF < 5%
- HWE-P-Value < 0.0001
- Remove samples with missing case/control criterion
- Sex check
- Remove samples and/or variants with high genotypic missing rate
- Imputation of missing values: IMPUTE2

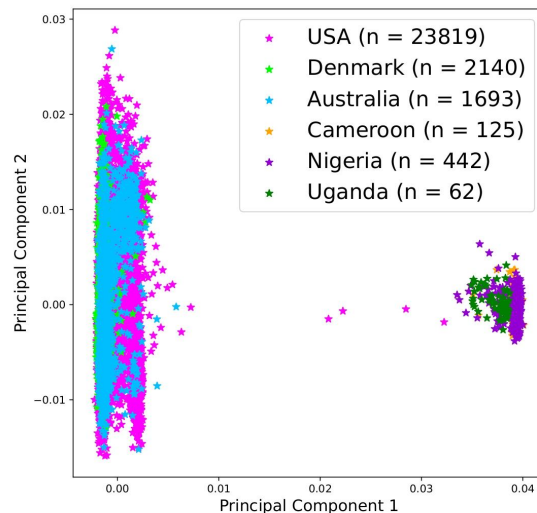
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- Subpopulations definition

Assign subpopulations in multitask framework according to PCA patterns by using K-means clustering.



**POP1:** USA, Denmark and Australia and **POP2:** Cameroon, Nigeria and Uganda

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- Evaluation

- **Validation using simulated data**

Generate simulations with specified multi locus disease model in specified regions

⇒ Compute **false positives rate**

- **Estimation of the stability of the selection** <sup>[1,2]</sup>

$$Stability = \hat{\Phi}(s_1, s_2, \dots, s_M) = \frac{1}{M(M-1)} \sum_i \sum_{j \neq i} sim(s_i, s_j)$$

- **Comparison with the state-of-the-art methods**

[1] Kuncheva et Al., A stability index for feature selection. 2008, *IASTED International Conference on Artificial Intelligence and Applications*.

[2] Nogueira et Al., On the Stability of Feature Selection Algorithms. 2018, *Journal of Machine Learning Research* 18.

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- Comparison with the state-of-the art methods**

- Adjusted Lasso: after PCA adjustment** for population stratification at the **SNP level**
- Adjusted group Lasso: after PCA adjustment** for population stratification at **LD-groups level**
- Stratified group Lasso** for each subpopulation at **LD-groups level**
- Stratified Lasso** for each subpopulation at **the SNP level**
- Adjusted GWAS: Classic GWAS** after PCA adjustment

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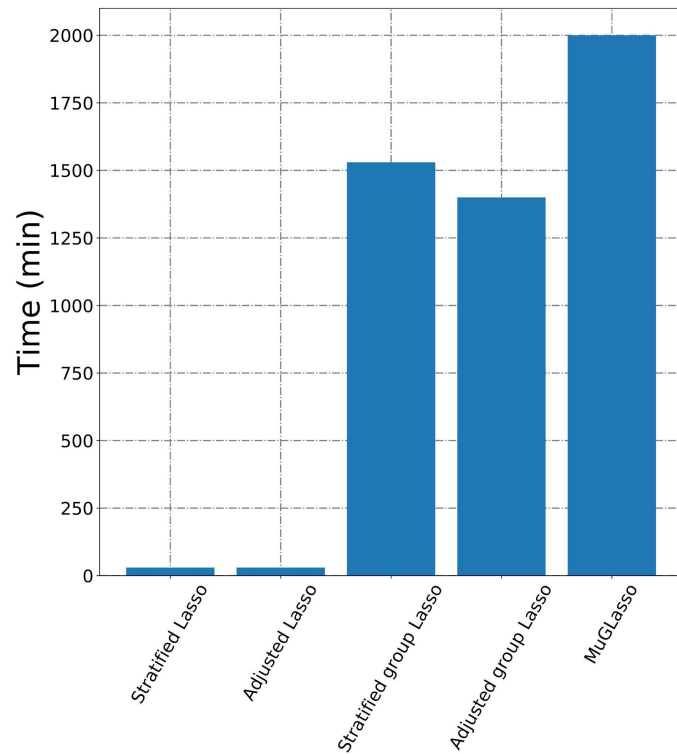
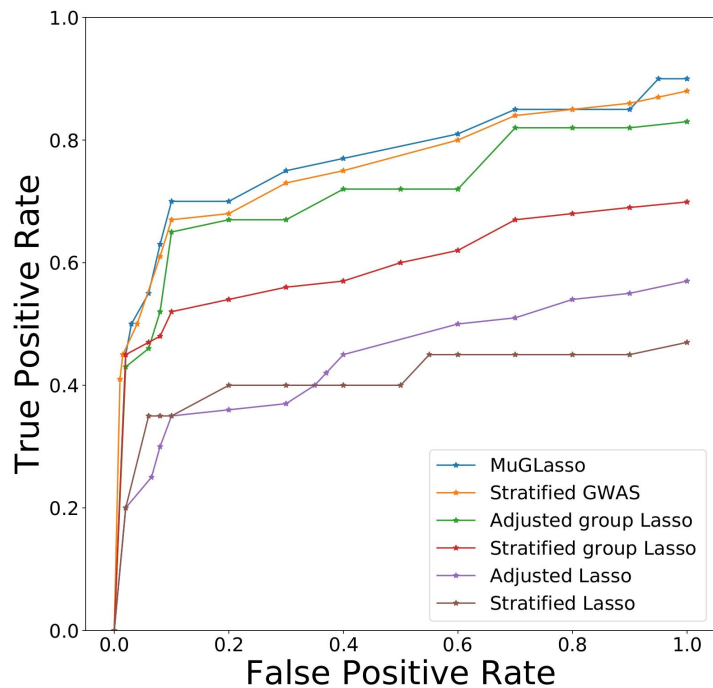
- **Comparison with the state-of-the art methods**
      - **Computational time**

[1] Kuncheva et Al., A stability index for feature selection. 2008, *IASTED International Conference on Artificial Intelligence and Applications*.

[2] Nogueira et Al., On the Stability of Feature Selection Algorithms. 2018, *Journal of Machine Learning Research* 18.

# Results of MuGLasso and comparison

Multitask group Lasso outperforms the state-of-the-art methods on simulated data

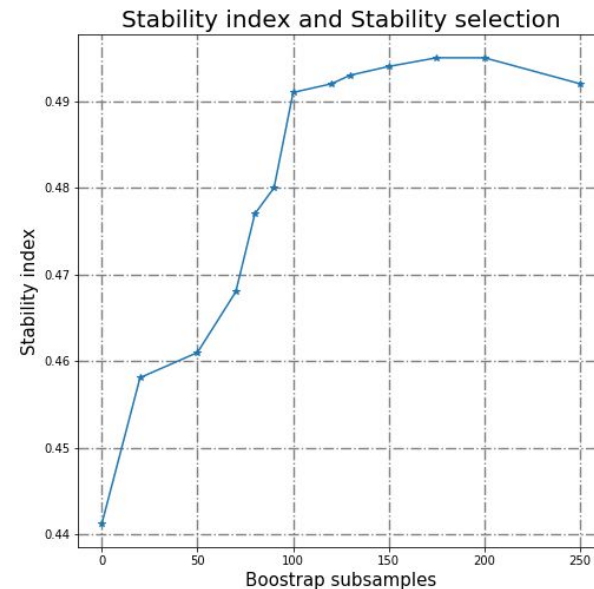


# MuGLasso results and comparison

Multitask group Lasso is **more stable** than the state-of-the-art methods.

**Simulated data:**  $n=4,000$  ;  $p=1,000,000$  ; LD-groups number = 35,792 groups

Methods	Number of selected LD-groups	Number of selected SNPs	Stability index	Selection level
<b>MuGLasso</b> (100 bootstraps)	5,623	155,312	0.4912	LD-groups
<b>Adjusted group Lasso</b>	6,054	162,104	0.4134	LD-groups
<b>Stratified group Lasso</b>	4,836	154,732	0.3398	LD-groups
<b>Adjusted Lasso</b>	5,379	158,856	0.2368	Single-SNP
<b>Stratified Lasso</b>	5,704	168,158	0.1742	Single-SNP
<b>Adjusted GWAS</b>	5,063	141,340	-	Single-SNP



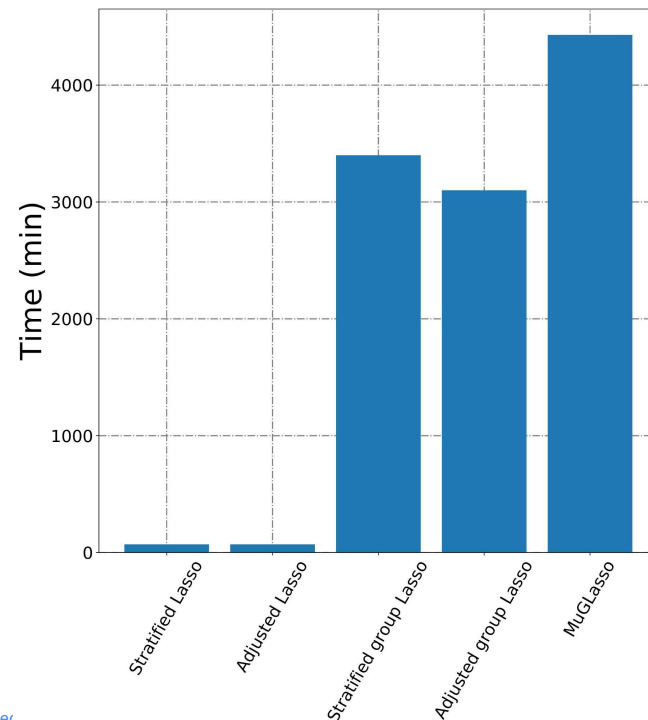
⇒ The feature selection at the LD-groups level alleviate the curse of dimensionality and the lack of stability.

# MuGLasso results and comparison

Multitask group Lasso is **more stable** than the state-of-the-art methods.

**Real data:** DRIVE Breast Cancer OncoArray<sup>[1]</sup> n=28,282 ; p=313,237 ; LD-groups number = 17,782 groups

Methods	Number of selected LD-groups	Number of selected SNPs	Stability index	Selection level
<b>MuGLasso</b> (100 bootstraps)	62	1,357	0.4312	LD-groups
<b>Adjusted group Lasso</b>	59	1,293	0.3234	LD-groups
<b>Stratified group Lasso</b>	58	1,119	0.2498	LD-groups
<b>Adjusted Lasso</b>	41	874	0.2068	Single-SNP
<b>Stratified Lasso</b>	38	789	0.1581	Single-SNP
<b>Adjusted GWAS</b>	16	306	-	Single-SNP



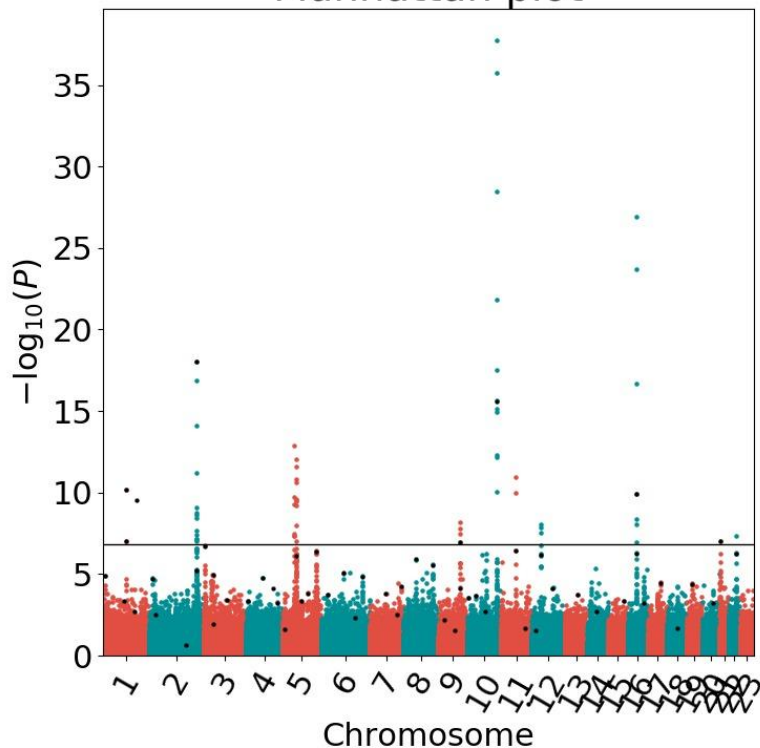
<sup>[1]</sup> DRIVE: "General Research Use" dataset in DRIVE Breast Cancer OncoArray Genotypes, available from dbGaP (study accession: phs001265/GRU), accesser



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Manhattan plot

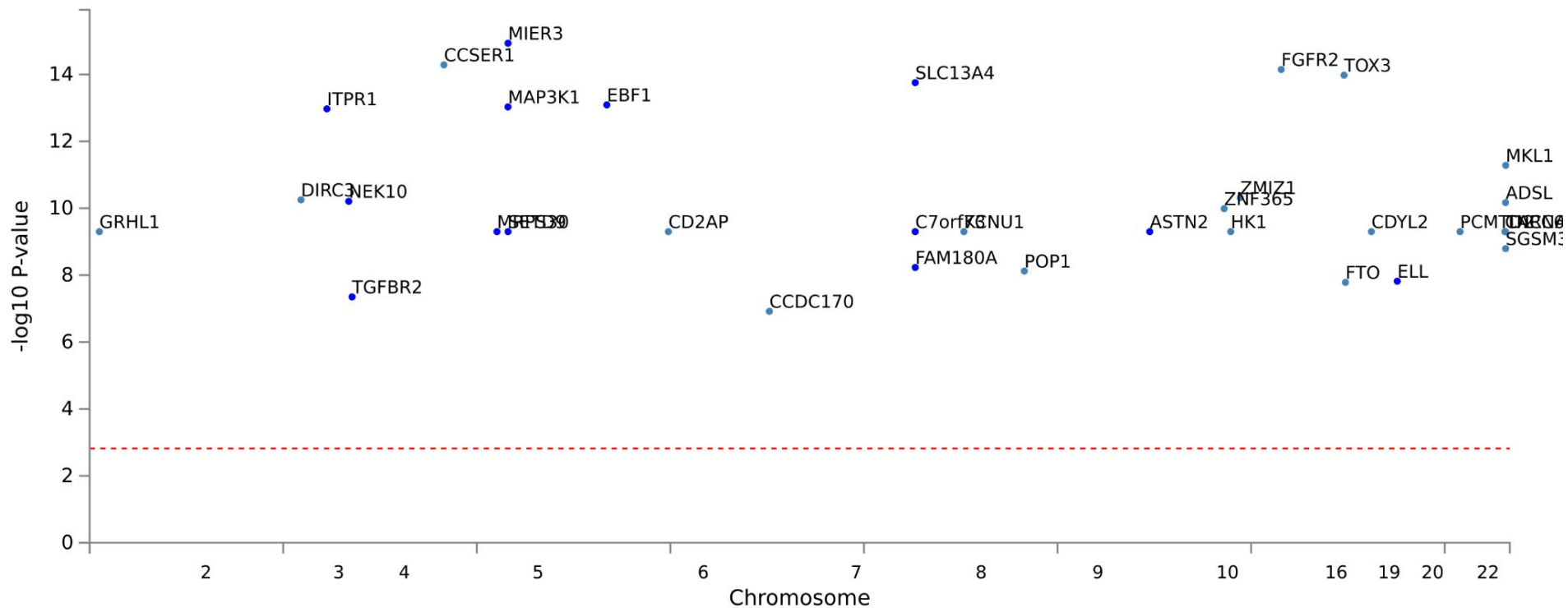


Black dots corresponds to Multitask group Lasso **discoveries**  
(one SNP per LD-group is represented in the Manhattan plot)

<sup>[1]</sup> DRIVE: "General Research Use" dataset in DRIVE Breast Cancer OncoArray Genotypes, available from dbGaP (study accession: phs001265/GRU), accessed under project #17707.

# Breast cancer risk loci detected by MuGLasso on DRIVE

Mapping of selected SNPs using MuGLasso to genes using FUMA<sup>[\*]</sup>



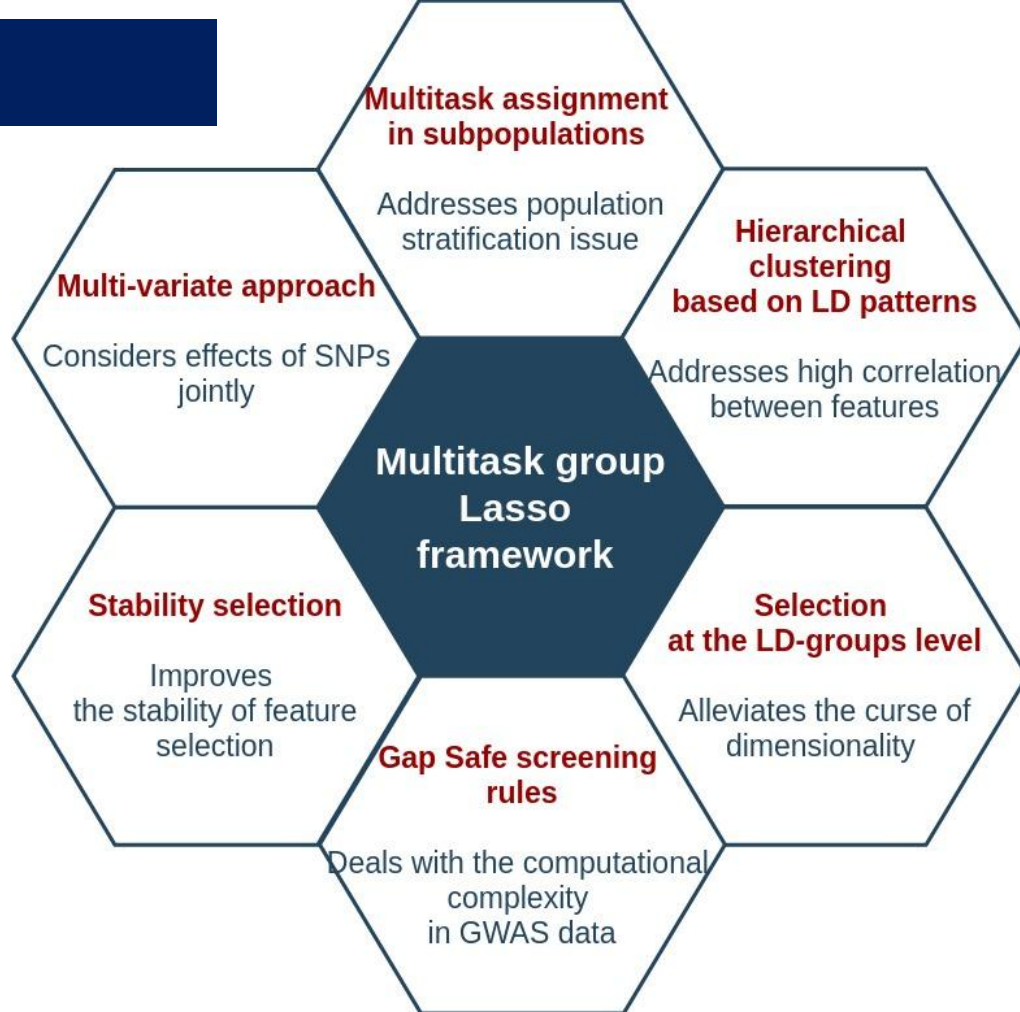
[\*] FUMA: <https://fuma.ctglab.nl/>

# Breast cancer risk loci detected by MuGLasso on DRIVE

- All SNPs/genes found by adjusted GWAS were also selected by MuGLasso.
- **9 genes** were discovered by adjusted GWAS and 32 genes were discovered by MuGLasso.
- **17 of 32 genes** had been previously identified by a meta-GWAS containing the DRIVE data.
- **7 genes** were found in the literature prior evidence of relationship with breast cancer risk or tumor growth.

Genes found by adjusted GWAS	<b>ITPR1, MRPS30, MAP3K1, SETD9, MIER3, EBF1, FGFR2, TOX3, MKL1</b>
Genes found by MuGLasso	<b>ITPR1, MRPS30, MAP3K1, SETD9, MIER3, EBF1, FGFR2, TOX3, MKL1, ADSL, ASTN2, C7orf73, CACNA1I, CCDC170, CCDC91, CCSER1, CD2AP, CDYL2, DIRC3, ELL, ESR1, FTO, GRHL1, HK1, HRSP12, KCNU1, LUC7L3, MED21, NEK10, NUP205, PAX9, POP1, PPFIBP1, PTHLH, REP15, SGSM3, SSBP4, TGFBR2, TNRC6B, ZMIZ1, ZNF365</b>
Genes discovered for subpopulation POP1	<b>ESR1, SGSM3, MED21, REP15</b>
Genes discovered for subpopulation POP2	<b>DIRC3, LUC7L3</b>

# Conclusion



# Acknowledgements

- CBIO (Mines ParisTech)
- GWAS team
- U900 (Institut Curie)
- This work was supported in part by Agence Nationale de la Recherche (ANR-18-CE45-0021-01 and ANR19-P3IA-0001)

**T**HANK **Y**OU!

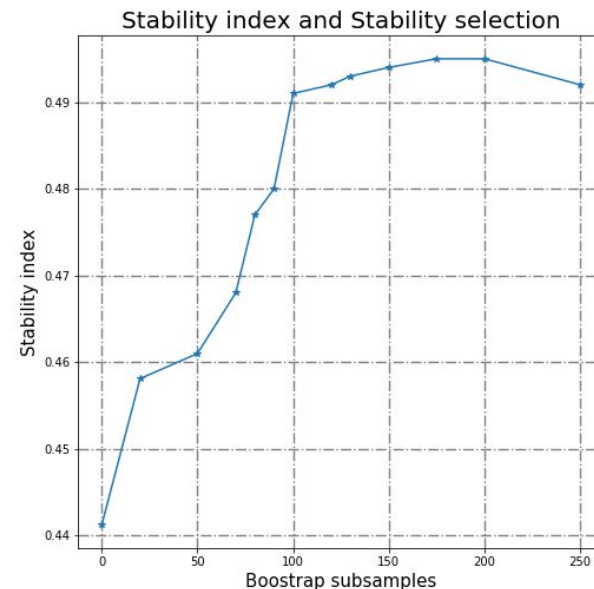


# MuGLasso results and comparison

Multitask group Lasso is **more stable** than the state-of-the-art methods.

**Simulated data:**  $n=4,000$  ;  $p=1,000,000$  ; LD-groups number = 35,792 groups

Methods	Number of selected LD-groups	Number of selected SNPs	Stability index	Selection level
<b>MuGLasso</b> (100 bootstraps)	5,623	155,312	0.4912	LD-groups
<b>Adjusted group Lasso</b>	6,054	162,104	0.4134	LD-groups
<b>Stratified group Lasso</b>	4,836	154,732	0.3398	LD-groups
<b>Adjusted Lasso</b>	5,379	158,856	0.2368	Single-SNP
<b>Stratified Lasso</b>	5,704	168,158	0.1742	Single-SNP
<b>Adjusted GWAS</b>	5,063	141,340	-	Single-SNP



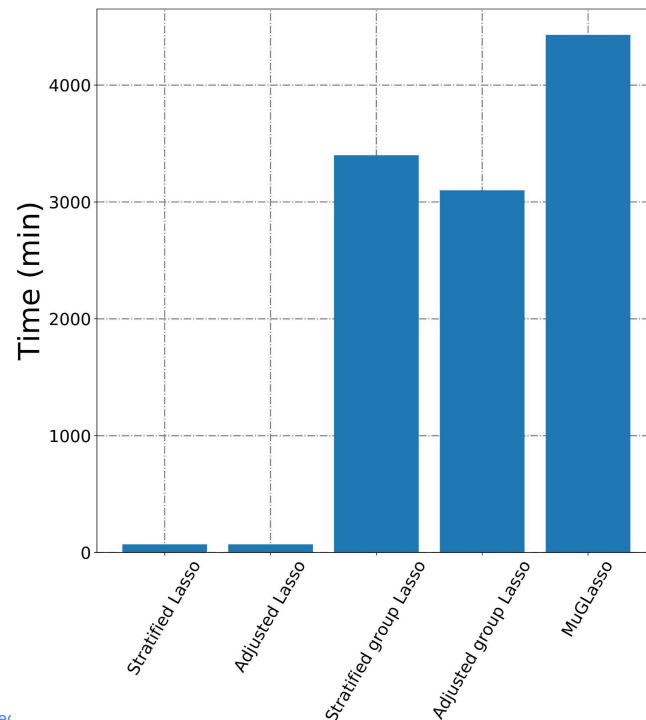
⇒ The feature selection at the LD-groups level alleviate the curse of dimensionality and the lack of stability.

# MuGLasso results and comparison

Multitask group Lasso is **more stable** than the state-of-the-art methods.

**Real data:** DRIVE Breast Cancer OncoArray<sup>[1]</sup> n=28,282 ; p=313,237 ; LD-groups number = 17,782 groups

Methods	Number of selected LD-groups	Number of selected SNPs	Stability index	Selection level
<b>MuGLasso</b> (100 bootstraps)	62	1,357	0.4312	LD-groups
<b>Adjusted group Lasso</b>	59	1,293	0.3234	LD-groups
<b>Stratified group Lasso</b>	58	1,119	0.2498	LD-groups
<b>Adjusted Lasso</b>	41	874	0.2068	Single-SNP
<b>Stratified Lasso</b>	38	789	0.1581	Single-SNP
<b>Adjusted GWAS</b>	16	306	-	Single-SNP



<sup>[1]</sup> DRIVE: "General Research Use" dataset in DRIVE Breast Cancer OncoArray Genotypes, available from dbGaP (study accession: phs001265/GRU), accesser